

Data-Driven Chance-Constrained Mixed Integer Nonlinear Bilevel Optimisation Via Copulas

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Motivation

- Optimise planning and scheduling under **multivariate, continuous-time product-demand uncertainty**;
- Set risk levels and production targets to enhance operational efficiency and **maximise demand satisfaction**;
- Enable informed decision-making under **correlated and interdependent demand patterns**.



Objective: Establish a **data-driven optimisation framework** that empirically constructs the joint distribution while capturing the dependence structure in the data.

Modelling Methodology

Given:

- (a) prescribed risk level
- (b) historical demand data

Determine:

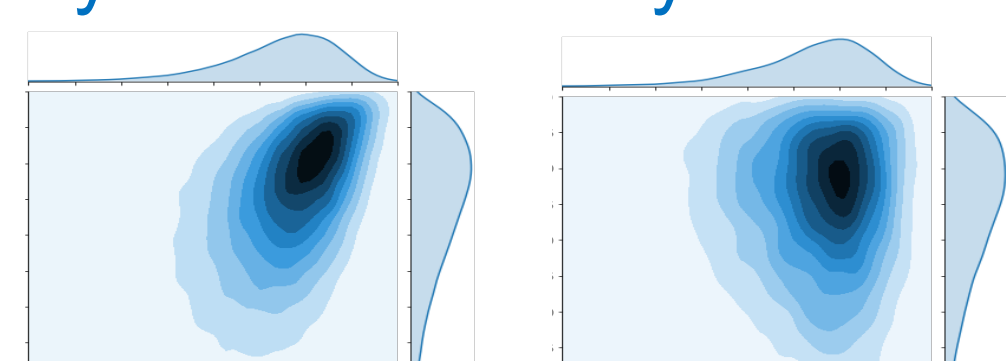
- ✓ multivariate dependency
 - ✓ demand distributions
 - ✓ production level
 - ✓ Inventory and backlog
- So as to maximise profit and product satisfaction rate

Copulas

→ Captures multivariate demand dependency with uncertainty

Data-driven coupling function $\mathcal{C}$ to join multiple marginal distributions into one multivariable distribution under certain dependence structures.

$$F(x_1, x_2) = \mathcal{C}_{12}(F_1(x_1), F_2(x_2))$$



Multivariate distributions with copula dependency (left) and without (right).

Individual Chance-Constrained Programming (ICCP)

→ Transforms uncertainties to quantitative risk levels

Optimisation-based method to enforce probabilistic constraints under specified risk threshold α :

$$P(x_i \geq D_i) \geq 1 - \alpha, \quad i \in [1, \dots, n]$$

x_i = estimated production level
 D_i = scenario-based demand
 α_i = predetermined risk level

DOMINO

→ Data-driven Optimisation of bilevel Mixed-Integer Nonlinear Framework under uncertainty

Data-driven hierarchical optimisation [1]

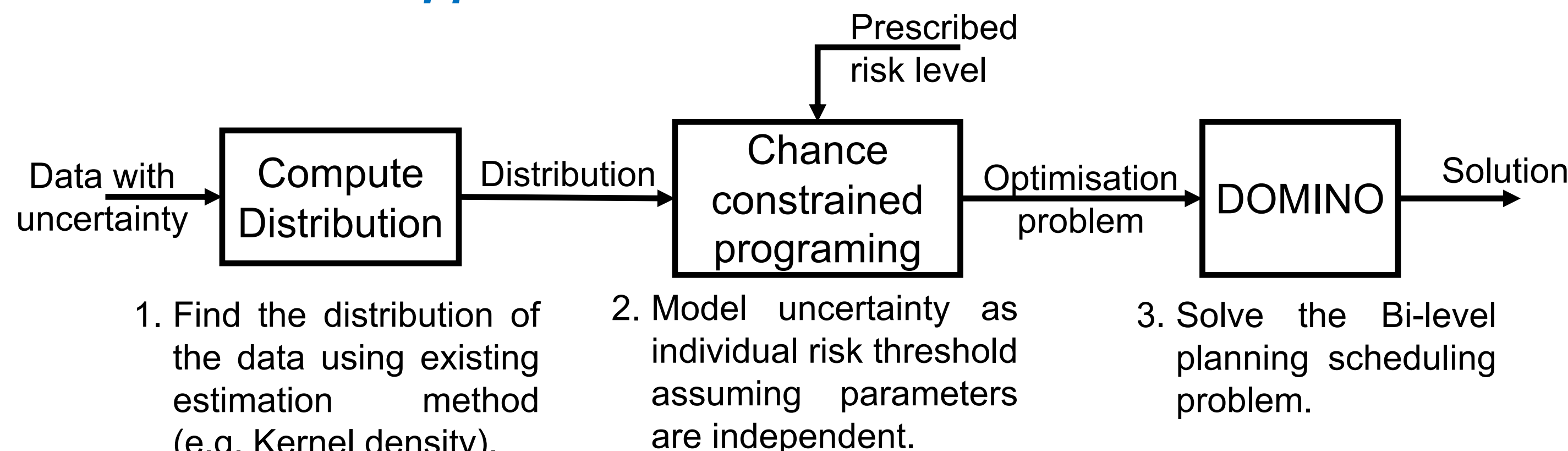
DOMINO converts bi-level MINLP problems into single-level grey-box optimisations, iterating steps until convergence [2].

- Step 1. Sampling upper-level decision variables
Sample planning targets to define candidate strategic scenarios.
- Step 2. Lower-level optimisation
Solve scheduling to *global optimality* using a deterministic solver.
- Step 3. Data-driven upper-level optimisation
Apply data-driven optimisation algorithm to identify optimal upper-level decisions.

Data-driven Optimisation Framework

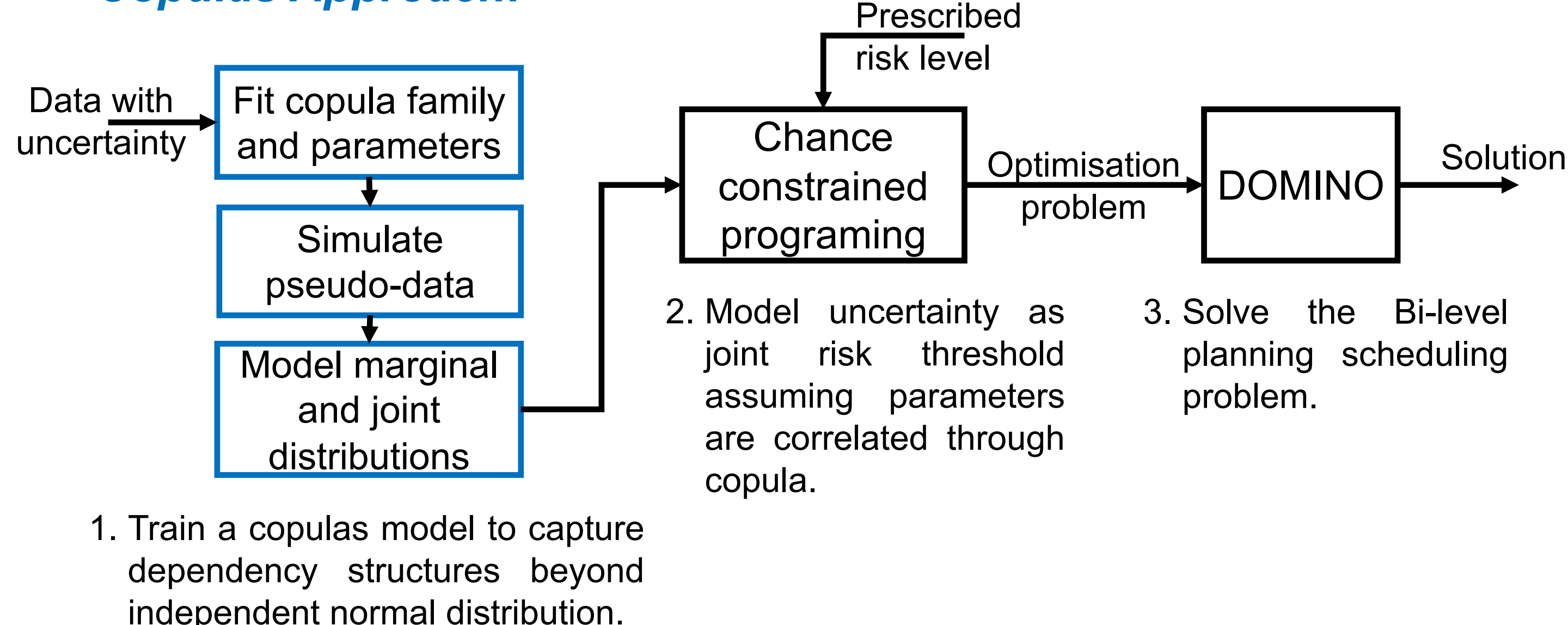
Data-driven Individual Chance-Constrained

Conventional Approach:



Data-driven Individual Chance-Constrained

Copulas Approach:



Monte Carlo Simulation

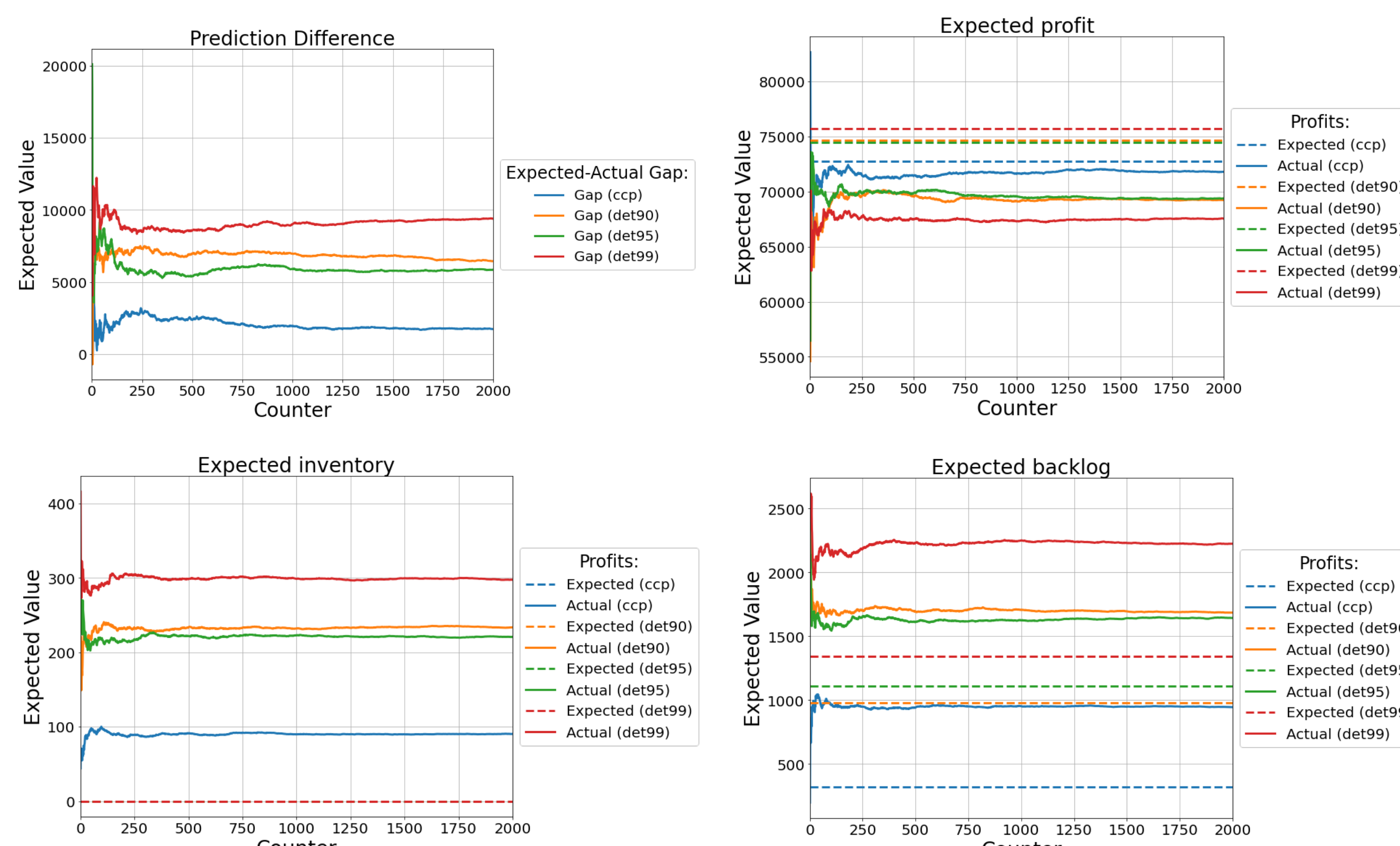
1 Training Phase (5,000 scenarios)

- Fit copula to capture demand correlations
- Estimate production targets & schedules
- Predict expected profit values (shown as dotted lines)

2 Evaluation Phase (2,000 scenarios)

- Solve planning and scheduling MINLP in [3]
- Test target robustness under varying actual demand levels
- Record realised profits (shown as solid lines)

➤ Compare robustness between **chance-constrained 95%-quantile copula** model vs **conventional ICC** model (90%, 95%, 99% confidence levels).

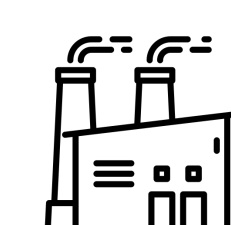


Observations for chance-constrained copula model:

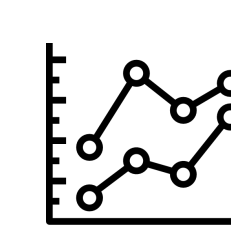
- ✓ **Highest realised profit** among all tested approaches
- ✓ **Minimal deviation** between expected (dotted) and actual (solid) profit curves
- ✓ **Stable outcomes** in extreme or uncertain conditions

Case Studies

Joint vs individual satisfaction level

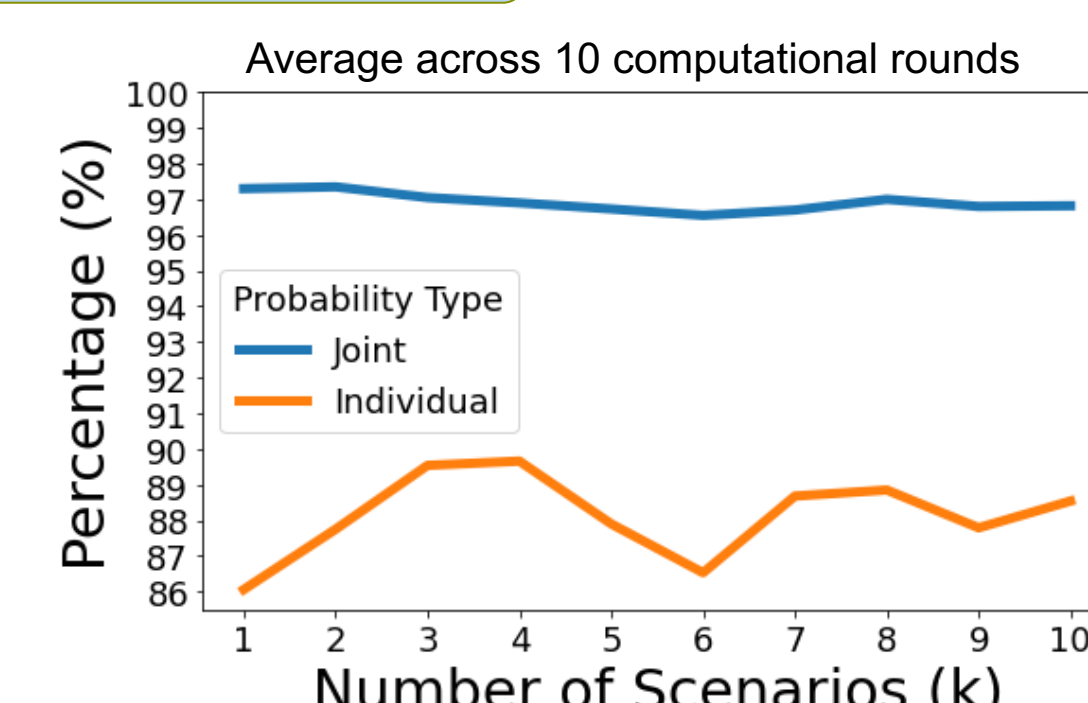


6 products
2 weeks



1k to 10k
scenarios

- ✓ Data-driven chance-constraint copula model achieves higher joint satisfaction rate
- ✓ Captures dependencies between product demands for more realistic forecasting

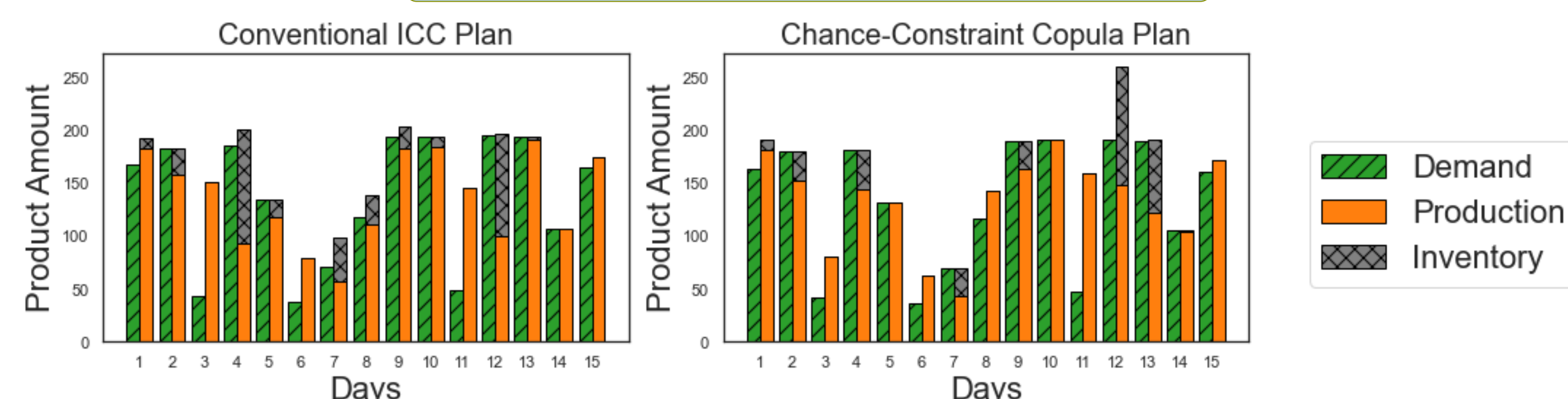


Performance comparison: copula vs conventional models

	Conventional ICC (99%)			Chance-Constrained Copulas (99%)		
Runs	Time (h)	Profit (k)	Samples	Time (h)	Profit (k)	Samples
Average	8.62	4.92	7668.6	10.25	4.87	8496.2
STD	1.79	0.46	1443.2	3.48	0.12	2625.4

- ✓ Slightly lower nominal profit due to conservative production and sales targets
- ✓ Reduces the likelihood of overproduction and excess inventory costs.

Production schedule comparison



- ✓ Copula-generated demands are lower, more aligned with actual market behaviour
- ✓ Observes lower inventory levels and fewer inventory-holding days

Conclusions

- Integrates data-driven copulas to **capture dependencies in multivariate production demand** in planning and scheduling problems.
- Ensures decisions remain feasible within a **specified risk threshold**.
- Chance-constraint copulas model identifies **robust production targets** that reduce inventory and backlog levels and **enhance joint satisfaction** across products.

Literature Cited:

- [1] Beykal, B., Avraamidou, S., Pistikopoulos, I. P., Onel, M., & Pistikopoulos, E. N. (2020). *Journal of Global Optimization*, 78(1), 1-36.
[2] Nikkhah, H., Aghayev, Z., Shahbazi, A., Charitopoulos, V. M., Avraamidou, S., & Beykal, B. (2025). *Digital Chemical Engineering*, 14, 100218.
[3] Charitopoulos, V. M., Dua, V., & Papageorgiou, L. G. (2017). *Industrial & Engineering Chemistry Research*, 56(39), 11186-11205.

Acknowledgements:

Financial support from UK EPSRC grants EP/V051008/1 and EP/W003317/1, and the donors of ACS Petroleum Research Fund Doctoral New Investigator Grant PRF# 66086-DNI9 are gratefully acknowledged.

